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Modelling and trajectory tracking of wheeled mobile robots

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Abstract

Differential drive mobile robots are widely used due to their simplicity, easiness of control and flexibility. This paper discuss a detailed modeling of a differential drive robot taking into account the kinematics, actuator dynamics and rolling resistances of the wheels. Controllers have been designed for smooth trajectory tracking. Different trajectories similar to real life scenarios have been created and the model and control algorithm are seen to give accurate trajectory tracking.

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Keywords: Differential Drive mobile robot; Kinematic controller; actuator dynamics; trajectory tracking

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1. Introduction

Autonomous mobile robots are finding widespread application is many areas like mining, space exploration and in service industry. The differential drive mobile robot (DDMR) is one such robot that has gained wide popularity due to its simplicity and ease of control [2]. The differential drive robot consists of two individually propelled wheels and a third wheel called castor wheel that can move freely in space. By adjusting the power applied to motors, the robot can be operated to go forward, rotate in place or perform movement on any arbitrary curve in plane.

Several research work on the modelling and control of such robots [1, 2, 3]. However most of these works handle the two aspects separately. Detailed modeling of the wheeled robot taking into account the actuator dynamics, rolling resistances and coupling constraints has not been attempted in most works when designing control schemes.

This paper discusses a detailed kinematic and dynamic model for the mobile robot which includes the chassis dynamics and actuator dynamics. Trajectory tracking is implemented using two controllers. A kinematic controller has been used for outer loop control to generate the reference velocities whereas a proportional

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controller has been used as inner loop controller to generate angular velocities for the wheels of the robot. Finally, several test trajectories were created and the simulation results are presented are seen to show excellent trajectory tracking.

2. Modeling of differential drive robot

The differential drive robot consists of a platform equipped with a front castor and a pair of rear co-axial drive wheels for isostatic equilibrium. Each of these drive wheels are independently driven by a DC motor which is in turn energized by a control voltage. By varying the power applied to the motors the differential wheeled mobile robot can be made to move in straight line or trace different trajectories like curves, circles etc. Deriving a precise mathematical model is a crucial part for designing any control system. Kinematic and dynamic model of the robot has been discussed below.

2.1. Kinematic modelling

Kinematic modeling deals with the geometric relationships that govern the system and studies the mathematics of motion without considering the affecting forces. For a differential drive which has two wheels with radius r paced at a distance L/2 from robot center. with θ be the robot orientation angle measured from the x-axis. From [1] and [2] the following equations are obtained:

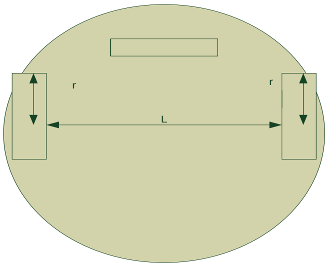

Fig. 1. Differential drive robot.

$$v = r\omega \tag{1}$$

$$w = \frac{d\theta}{dt} \tag{2}$$

The speed of each wheel in the robot frame is $r\omega$, therefore the translational speed in the robot frame is the average velocity

$$v_{mob} = \frac{r(\omega_R + \omega_L)}{2} \tag{3}$$

whereas the rotational velocity is given by $\omega = \frac{v_R - v_L}{L}$. (4)

The mapping between the inertia frame and the robot frame is done through the standard orthogonal transformation. Hence the robot velocity in the inertial or global frame is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r(\omega_R + \omega_L)}{2} \\ 0 \\ \frac{r(\omega_R - \omega_L)}{2} \end{bmatrix} \tag{5}$$

2.2. Dynamic modelling

Dynamic modelling of the robot is the study of motion in which forces and energies are modeled and studied. The actuator modeling is required to find the relationship between the control signal and the mechanical system input.

2.2.1 Dynamics of the actuator: The actuating machines mostly used in mechatronics motion control systems are DC machines (motors). The mobile platform motion control can be simplified to a DC motor motion control. In modelling DC motors and in order to obtain a linear model, the hysteresis and the voltage drop across the motor brushes are neglected, the motor input voltage, Vin is applied to the field or armature terminals.

The open loop transfer function of the dc motor [2] is given by

$$\frac{\omega_m(s)}{V(s)} = \frac{k_t / n}{\left[(L_a J)s^2 + (R_a J + BL_a)s + (R_a B + k_t k_b)\right]} \tag{6}$$

where R_a -resistance and L_a -inductance of the motor
J –moment of inertia and B –viscous coefficient referred to the motor shaft
k_t -torque constant ,k_b -emf constant and n-gear ratio.

The left and right wheel motors are modelled using the above equation.

2.2.2 *Dynamics of the chassis*: Two methods are used for dynamic mode derivation.-the Newton Euler method and the Lagrange method. The lagrange method has been chosen here due to its more systematic nature and automatic elimination of workless and constraint forces.

From the lagragian equations of motion [6], the following relations are obtained*:*

$$\dot{v} = \frac{(T_L + T_R)}{m} - k_v v \tag{7}$$

$$\dot{\omega} = \frac{(T_L + T_R)l}{r} - k_\omega \omega \tag{8}$$

Where T_L and T_R represent left and right motor torques, k_v-resistance coefficient against linear motion kω-resistance coefficient against rotary motion. The combined dynamic model of actuator and chassis is shown in Fig.2.

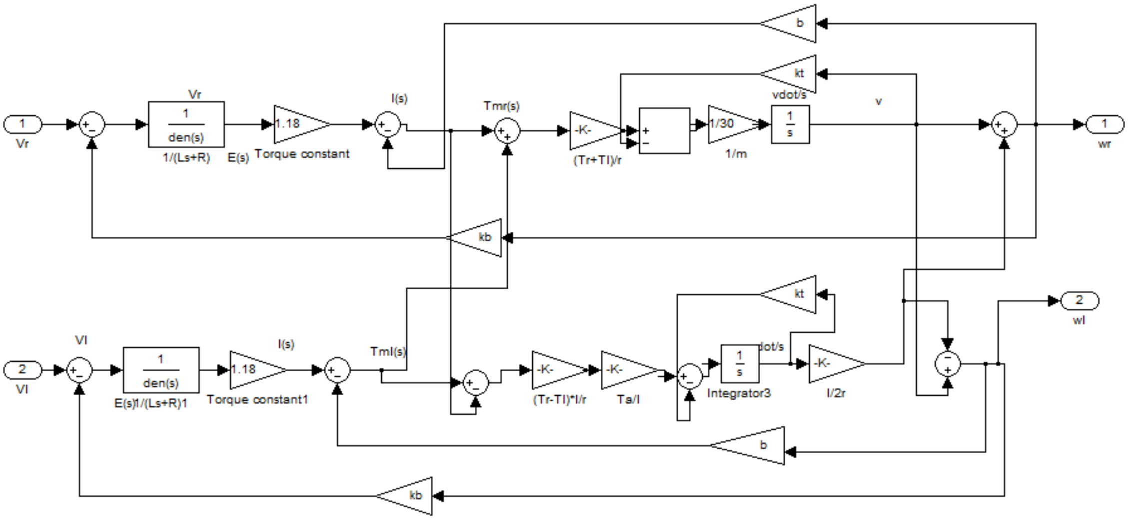

Fig.2 Dynamic model of actuator and chassis

3. 3. Trajectory tracking of DDMR

Trajectory-tracking problem is posed as follows[1]: It is assumed that the main robot has the posture: $P = (x, y, \phi)T$, and that the reference robot (to be followed) has the posture: $Pr = (xr, yr, \phi r)T$. The objective is to find control laws for the linear and angular velocities (v, w) of main robot such as: $\lim t\to\infty |x(t)-xr(t)| = 0$, $\lim t\to\infty |y(t)-yr(t)| = 0$, $\lim t\to\infty |\phi (t)-\phi (t)| = 0$.

Fig.3 shows the block diagram for trajectory tracking. For trajectory tracking and control loop has to be established that will provide as output the required input voltage for the wheels. There are two controllers in the control scheme. The outer controller is a kinematic controller that takes as input the errors in the x and y coordinate's between the reference position and actual position obtained from sensors along with the orientation of the robot. The kinematic controller gives the reference linear and angular speeds.

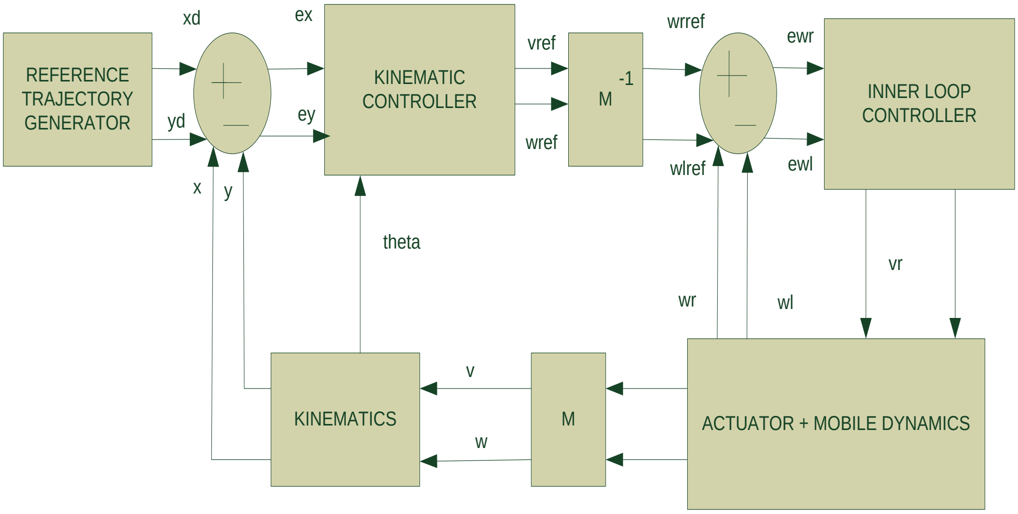

Fig.3 Block diagram for closed loop control

The kinematic controller (outer loop) used in the study [5] is given by:

$$\begin{bmatrix} v_{ref} \\ \omega_{ref} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \dfrac{-1*\sin\theta}{a} & \dfrac{-1*\cos\theta}{a} \end{bmatrix} \begin{bmatrix} \dot{x}_d + l_x \tanh\dfrac{k_x}{l_x}(x_d - x) \\ \dot{y}_d + l_y \tanh\dfrac{k_y}{l_y}(y_d - y) \end{bmatrix} \tag{9}$$

A simple P controller is used as the inner loop controller.

The following transformation has been used to change from linear and angular velocities to angular velocities of the wheels.

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = [M] \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \tag{10}$$

$$M = \begin{bmatrix} r/2 & r/2 \\ r/l & -r/l \end{bmatrix} \tag{11}$$

4. 4. Simulation of trajectory tracking of DDMR

The control scheme for trajectory tracking given in fig.4 was implemented in MATLAB Simulink. Different behaviors that need to be executed by a mobile robot like moving to a point, tracking a circle, tracking a line etc. were checked by generating the required trajectories.

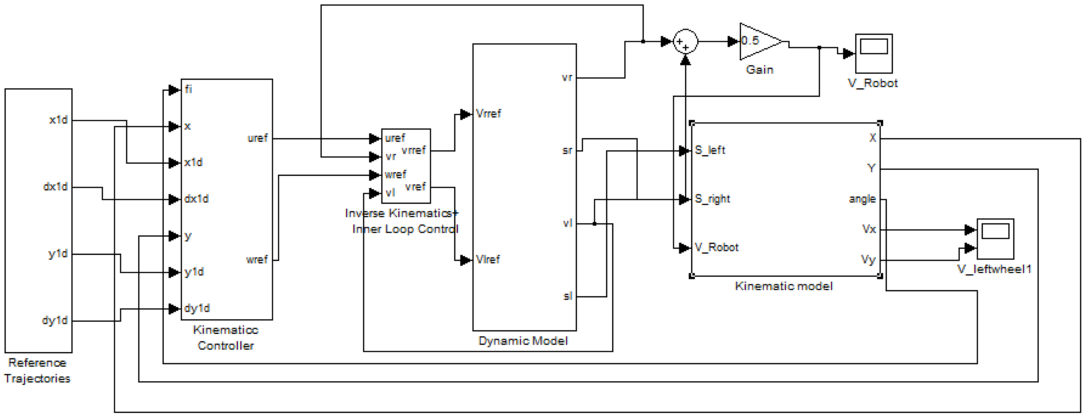

Fig.4. Simulation diagram for trajectory tracking

4.1 Simulation Results:

Case 1: Circular trajectory tracking of the robot.

As seen in Fig.5 the robot trajectory follows the reference circular trajectory of radius 1 cm. The robots initial position is at (0,0).

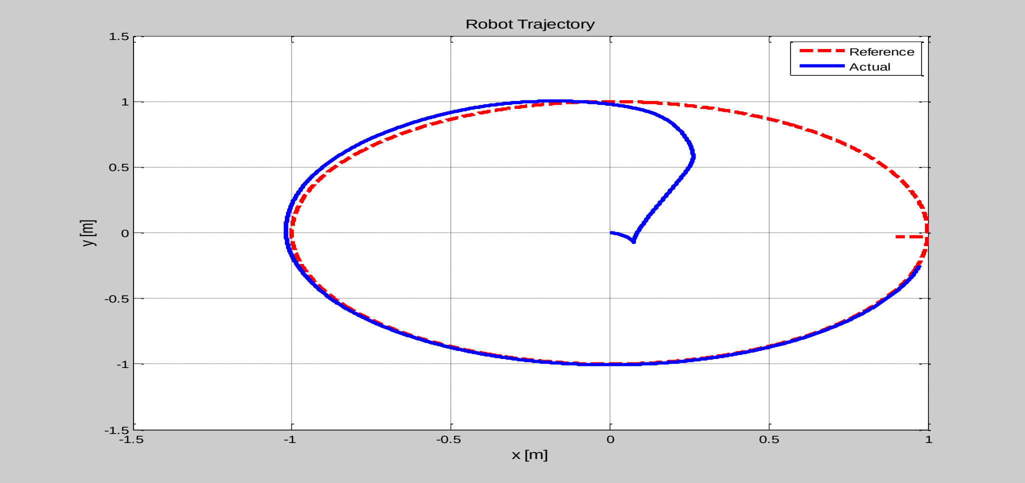

Fig.5 Circular trajectory tracking of mobile robot

Case 2: Straight line trajectory tracking of the robot.

Two straight lines with equation x=3 and one with unity slope are created. In both the cases, the robot moves from its initial position (0,0) and tracks the given straight line trajectory as shown in Fig.6.a. and Fig.6.b.

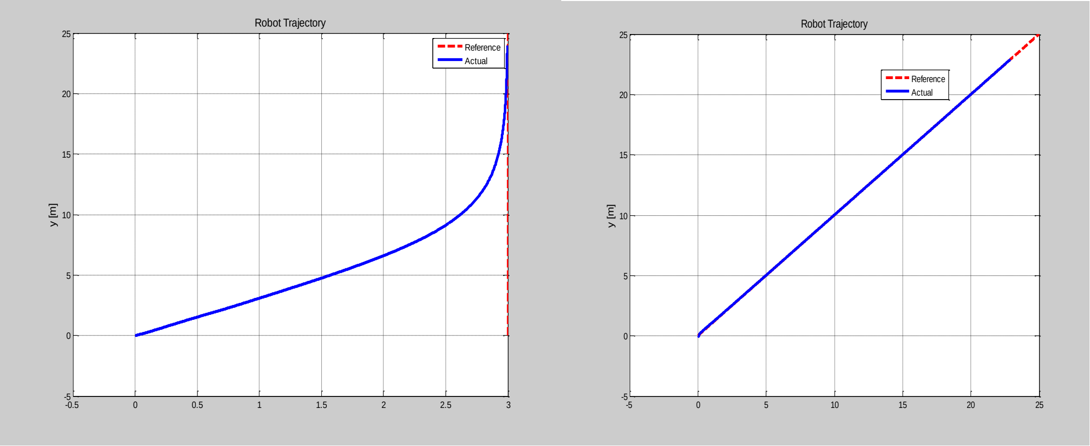

Fig.6.a. and b. Straight line trajectory tracking of the robot

Case 3: Moving to a point

The goal of the robot was chosen as (2, 3). The robot moves from its initial position (0,0) and reaches the desired position as in Fig.7.

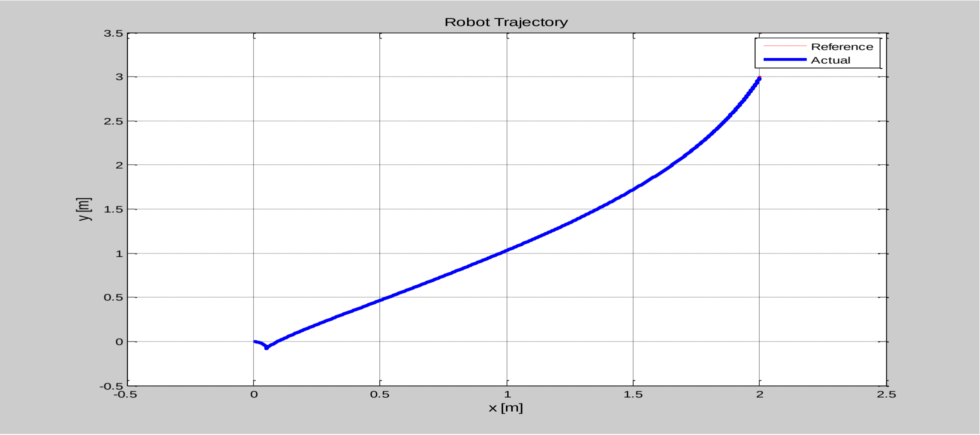

Fig.7 Robot moving to a desired position

Fig.s 8(a),(b) and 9 shows the error in x and y coordinates while the robot executed the behaviors: moving to a point, circular trajectory tracking and straight line trajectory tracking respectively.

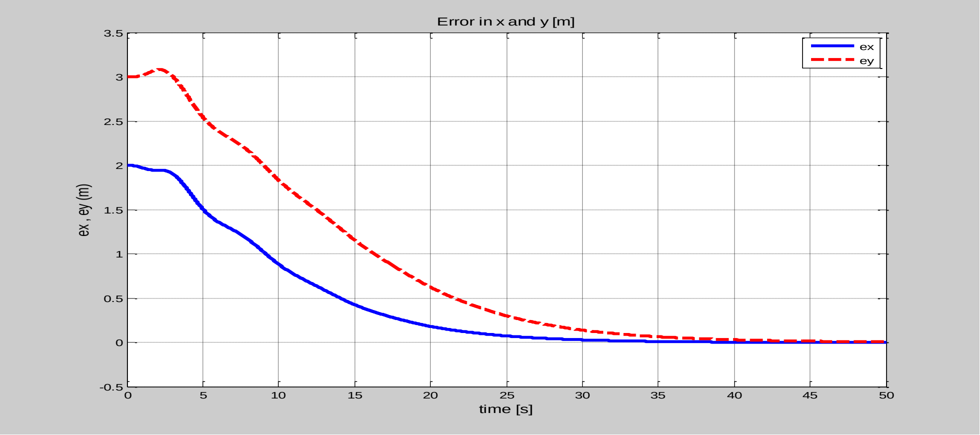

Fig.8 (a) Error in x and y-moving to a point

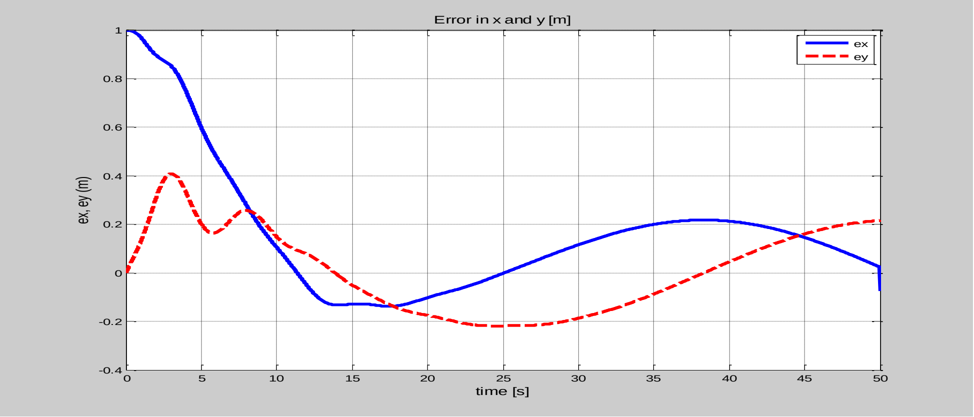

Fig.8(b) Error in x and y-circular trajectory tracking

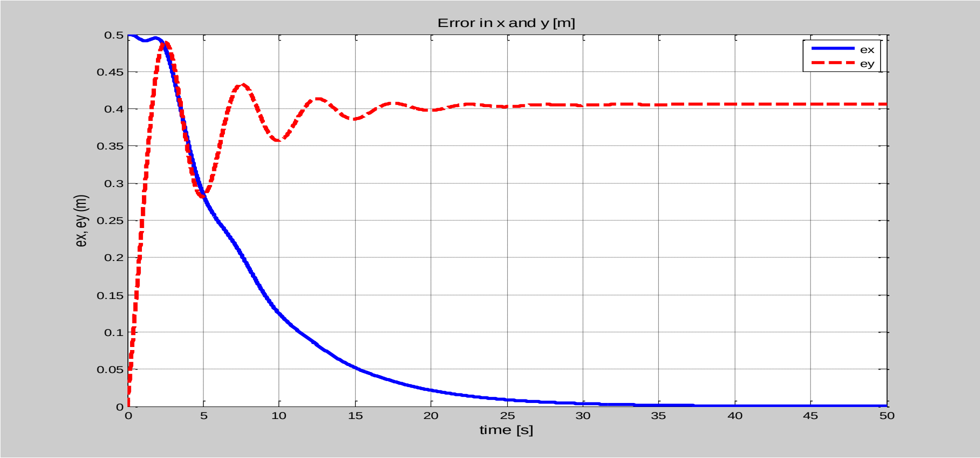

Fig.9 Error in x and y – straight line tracking

It was seen that for the same value of controller parameters, different trajectories have an influence on errors in the x and y co-ordinates.

Summary

A kinematic and dynamic mode has been created for the differential drive mobile robot using transfer function approach. A kinematic and simple dynamic controller was also done. The model with the control schemes has been able to satisfactorily track the given trajectory. The control scheme is good enough for basic tracking problems. More robust control algorithms will be studied in future.

References

[1] Egerstedt, M., 2013. Control of mobile robots.
[2] Ahmad A. Mahfouz, Ayman A. Aly, Farhan A. Salem, 2008, Mechatronics Design of a Mobile Robot System, IJISA, vol.5, no.3, pp.23-3,2013.
[3] Nwe, A. A., Aung, W. P. and Myint, Y. M. Software implementation of obstacle detection and for wheeled mobile robot avoidance system. World Academy of Science, Engineering and Technology, 42, 572–577.
[4] J. Craig,1989, Introduction to Robotics: Mechanics and Control,Addison-Wesley.
[5] Martins, F. N., Celeste, W. C., Carelli, R., Sarcinelli-Filho, M. and Bastos-Filho, T. F.,2008, An Adaptive Dynamic Controller for Autonomous Mobile Robot Trajectory Tracking. Control Engineering Practice, v. 16, p. 1354–1363.
[6] Ali Gholipour, Yazdanpanah.M.T., Dynamic tracking control of Nonholonomic Mobile Robot with Model Reference Adaptation for Uncertain parameters, European control Conference,2003, pp.3188-3122,2003.